PART VIII – COULOMB BARRIER AND THE BOHR RADIUS

Part VIII focuses on the Coulomb Barrier for hydrogen–hydrogen (H–H) fusion, explores the Quantum Wavespace Theory (QWST) approach to deriving it, and relates the results to phenomena at both the nuclear and atomic-scale such as the Bohr radius and ionization energies.

VIII.1 – Coulomb Barrier

Selected Investigation - Coulomb Barrier

Standard physics equation for the Coulomb barrier for H-H:
From standard physics, the potential energy (Coulomb barrier) that must be overcome to bring two protons close enough to fuse—without quantum tunneling—is:

$$E_C = \frac{k_e e^2}{r}$$

where:

- $k_e = 8.98755 \times 10^9 \ \mathrm{N{\cdot}m^2/C^2}$
- $e = 1.602177 \times 10^{-19} \ \mathrm{C}$
- $r = 1.0 \ fm \ = \ 1 \ \times 10^{-15} \ m$ distance between nucleons (often taken as $1 \ fm$)

This gives us the result

$$E_C = \ 2.30705 \times 10^{-13} \ Joules$$

and converting to MeV give us ($1MeV = 1.602177 \times 10^{-13} J$):

$$E_C = \ 1.44 \ MeV$$

This is the commonly cited H-H Coulomb barrier for a $1 \ fm$ proton-proton separation.

QWST equation for the Coulomb barrier for H-H:
Under Quantum Wavespace Theory (QWST), we seek to express the Coulomb barrier in terms of fundamental QWST constants. Specifically, we utilize the QWST-derived expression for e^2:

$$e^2 = \frac{A P_0 r_0^4}{\pi \alpha^{-1}(C^2 \cdot 10^{-7})}$$

where:

- $A = \frac{16(\pi^2 - 8)}{3\pi^2} \approx 1.010296$ (dimensionless wave geometry constant)
- $P_0 = 5.158515 \times 10^{35} \ \mathrm{Pa}$ (maximum stable energy density in nucleon core)
- $r_0 = 6.607241 \ \times 10^{-16} \ \mathrm{m}$ (radius of nucleon)

- $\alpha^{-1} = 137.036$ (inverse fine structure constant)
- $C^2 \cdot 10^{-7}$ = (SI-based definition of k_e)

Substituting this into our expression for E_C, we have:

$$E_C = \frac{k_e}{2r_0}\left(\frac{AP_0 r_0^4}{\pi\alpha^{-1}(C^2 \cdot 10^{-7})}\right)$$

Recall the identity for the Coulomb constant in SI units:

$$k_e = \frac{1}{4\pi\epsilon_0} = C^2 \times 10^{-7}$$

Thus, k_e cancels the $C^2 \times 10^{-7}$factor, simplifying the equation to:

$$E_C = \frac{AP_0 r_0^4}{(r_0)2\pi\alpha^{-1}}$$

This is the final simplified form of the Coulomb barrier energy in terms of QWST fundamental constants. We previously derived α^{-1} based on wave geometry, so we will retain that constant for simplicity. Finally, inserting known QWST values we obtain:

$$E_C = \frac{2.3075 \times 10^{-28}}{(r_0)2}$$

- If we set $r_0 = r = 1 \times 10^{-15}\ m$ we will recover the standard $1.44\ MeV$ result.
- Instead, we will use the QWST value $r_0 \approx 0.66\ fm$ then our result becomes:

$$E_C = 1.746 \times 10^{-13}\ \mathrm{J} = 1.0897\ MeV$$

We will use this result in our QWST derivations.

Coulomb Barrier in Terms of Multiple Shells (N)

We wish to determine the energy contribution at a specific "shell" ("reaction shell" or "reaction volume") of the nucleon wave structure. We can number each shell starting from the center of the nucleon $N = 0$ and moving in increments of N radially outward. We therefore reinstate N in the Coulomb barrier equation to derive an expression for energy E_R at a given separation distance $R = Nr_0$:

$$E_R = \frac{A\,P_0\,r_0^4}{\pi\,(Nr_0)\,\alpha^{-1}} = \frac{1}{N} \times \left(\frac{P_0\,A\,r_0^4}{\pi\,(r_0)\,\alpha^{-1}}\right)$$

This conveniently simplifies to:

$$E_R = E_C/N$$

Thus, we establish a relationship between the local pressure (energy density) and the separation distance between protons. Typical fusion reactions are $\sim 0.1\ MeV$ so we can estimate that quantum tunneling can occur at the "reaction shell" found at $N = 10$ (in increments of r_0). This provides us with additional detail of the wave geometry at the nuclear scale, as conceptualized in the following illustration:

Selected Investigation – Pressure Across Nucleon Shell Surfaces

We can also derive the pressure decrease as a function of N. The energy in the core, E_N, is also the same for each shell volume. However, since each shell volume increases with N, the pressure will decrease as we extend further from the core. We will integrate the maximum wave pressure P_m in increments of $2r_0$. To avoid confusion, we will use N_2 where $R = N_2 2r_0$ We take the integral of ½ a cosine curve, integrating from node to node $(-R_n to + R_n)$ to determine the pressure at even integers of N.

$$(nucleon\ core\ energy) = E_N = \frac{3}{2} A P_0 r_0^3 = \int_{R_N^-}^{R_N^+} 4\pi\left(R_N^{\pm}/\lambda\right)^2 P_m \cos\frac{\pi\lambda}{2r_0}\, dR$$

$$\frac{P_0}{P_m} = \frac{64N_2^2}{3A} \pm 2\pi N_2 B + 1$$

We use the average pressure P_{N2}, which cancels the $\pm 2\pi N_2 B$ term, and solve for P_{N2}:

$$P_{N2} = \frac{P_0}{(64N_2^2/3A+1)}$$

We can write the equation in terms of N where $R = Nr_0$ however odd values of N should be ignored, since the pressure is zero at the nodes.

$$P_N = \frac{P_0}{(16N^2/3A \ + \ 1)}$$

In practice, the $\pm 2\pi N_2 B$ term will be small compared to N_2^2 so this will have a negligible effect. gives us the energy–pressure relationship at separation $r = N_2 2r_0$

Table of Energy and Pressure for Reaction Shells at N

N	Description	$E_R(MeV)$	$P_N(Pa)$	r
0	Core (C-sphere)	$infinity$	5.158×10^{35}	0
1	Coulomb Barrier	1.08970000	8.216×10^{34}	0.6607×10^{-15}
2	Nucleon Contact	0.54485000	2.332×10^{34}	1.321×10^{-15}
4	Reference Pressure	0.27242500	6.036×10^{33}	2.643×10^{-15}
6	Reference Pressure	0.18161667	2.700×10^{33}	3.964×10^{-15}
8	Reference Pressure	0.13621250	1.522×10^{33}	5.286×10^{-15}
10	Fusion Reactor	0.10897000	9.753×10^{32}	6.607×10^{-15}
11760	Pressure balance (Rydberg Derivation)	$92.66\ eV$	7.066×10^{26}	7.770×10^{-12}
80000	Bohr's radius	$13.62\ eV$	1.527×10^{25}	5.286×10^{-11}

It is instructive to compare typical pressures and separation distances encountered in various physical scenarios, from nucleon cores up to laboratory-scale fusion devices:

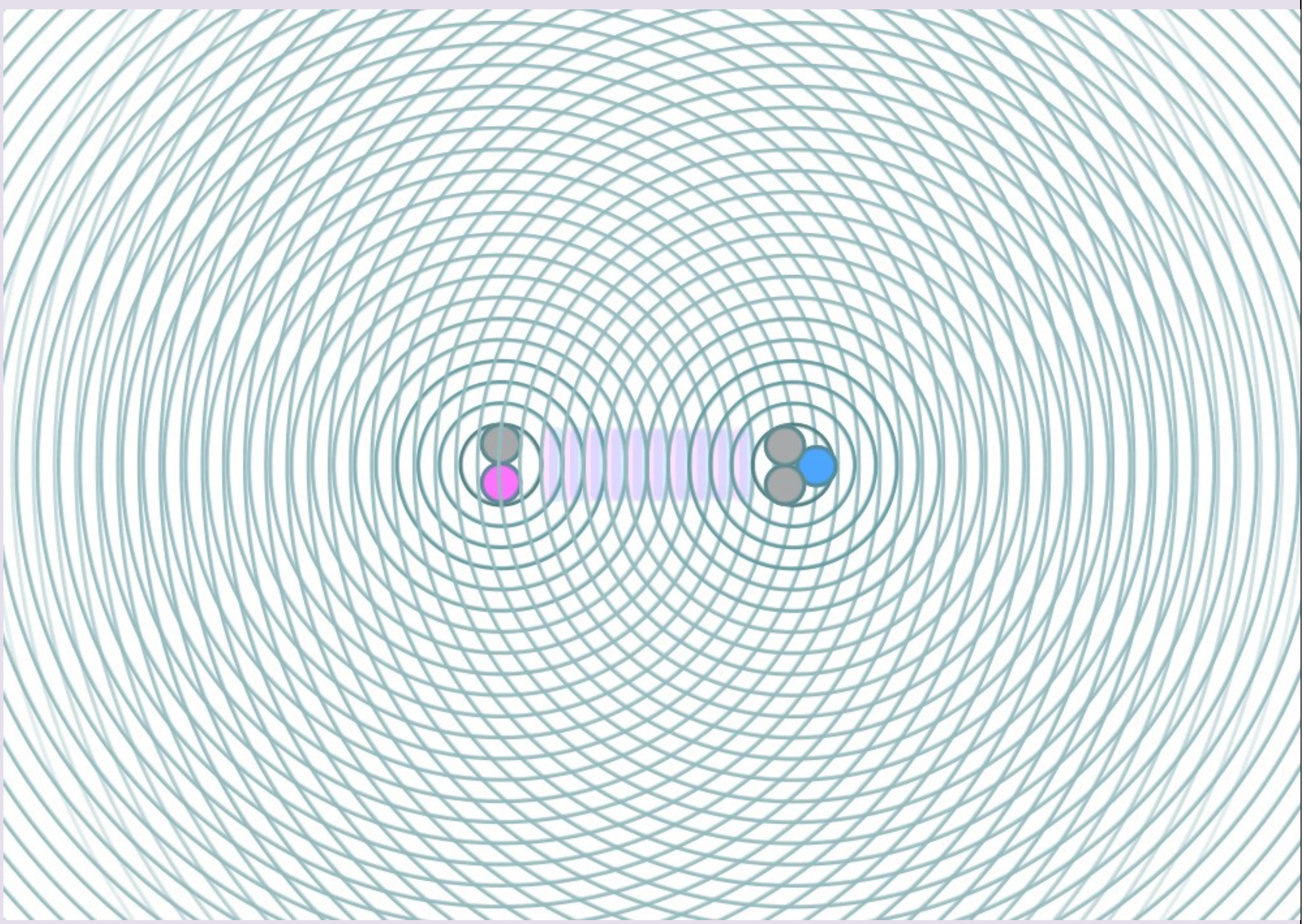
Table of Empirical Energy and Pressures at Nuclear Scales

Key Location	Nuclear-Scale Pressure (Pa)	Energy	N (R = Nr_0)	Distance R (fm)	Notes / Explanation
Coulomb Barrier (QWST)	8.216×10^{34}	$1.0897\ MeV$	1	$0.6607\ fm$	QWST theoretical minimum barrier
Balanced P approx. $3g \times 4$	7.066×10^{26}	$92.66\ eV$	11760	$9414.4\ fm$	Original QWST approximation
Bohr's Radius (a_0)	3.93×10^{25}	$13.6\ eV$	66137	5.286×10^{-11} m	Hydrogen atom electron ground state
QWST Core (P_0)	5×10^{35}	E_N	0	0	Fundamental QWST nucleon energy density
Empirical Fusion (Tokamak)	$10^{26} - 10^{27}$	$\sim 0.1\ MeV$	$> 10^4$	$> a_0$	Local nuclear pressure between nuclei at fusion temperatures
Empirical Fusion (Stellar Core)	$10^{27} - 10^{28}$	$\sim 0.01 - 0.1\ MeV$	$> 10^4$	$> a_0$	Nuclear-scale pressure at stellar fusion conditions
(NIF Empirical) ICF Fusion	$10^{28} - 10^{30}$	$\sim 0.01 - 0.1\ MeV$	$> 10^4$	$> a_0$	Nuclear-scale pressure during inertial fusion implosion
Particle Accelerator Collision	$10^{33} - 10^{35}$	$> 1000\ MeV$ $(GeV - TeV)$	$2(1.59) \sim 3r_0$	$< 2.544\ fm$	High-accelerator-energy collisions

Note that these are local, *nuclear-scale* values, not representing macro-scale conditions.

Thus, we establish a relationship between the local pressure (energy density) and the separation distance between protons. Typical fusion reactions are $\sim 0.1\ MeV$ so we can estimate that quantum tunneling can occur at $N = 10th$ "reaction shell" (with each shell being $2r_0$).

This provides us with additional detail of the wave geometry at the nuclear scale, as conceptualized in the following illustration:



A conceptual diagram of a D-T fusion reaction, showing the reaction volume that is created as the kinetic energy of motion is transferred to the stored energy of the nucleons, as the nucleons attempt to stabilize their energy. The nucleons are $20r_0$ or $N = 10$ apart, which is an estimate of the location achieved in fusion reaction where quantum tunneling can be achieved within a certain probability.

VIII.2 – Bohr's Radius (a_0)

Selected Investigation - Bohr Radius with QWST Relationships

We examine the derivation of the Bohr radius ($a_0 \approx 0.529$ Å) using fundamental QWST constants. This derivation relates a_0 to the maximum stable energy density P_0, and wave-geometry factors, including the nucleon radius r_0.

$$a_0 = \left(\frac{P_0}{e}\right)\left(\frac{A\, r_0^2}{2\pi}\right)^2$$

- $e = 1.602177 \times 10^{-19}\ C$ (static electron charge)
- $r_0 = 6.60724 \times 10^{-16}$ m (nucleon radius, 1/4 wavelength of the basic frequency)
- $P_0 = 5.15851475432 \times 10^{35}\ Pa$ (the maximum stable energy density of quantum wavespace as well as within the nucleon C-sphere)
- $A = 1.010296 = \frac{16(\pi^2-8)}{3\pi^2}$ (wave geometry constant, simplifying placeholder)

$$a_0 = 5.2918 \times 10^{-11}\ m$$

Thus, we have precisely derived the Bohr radius.

$$a_0 = N_a\,(r_0) \text{ or } N_a = a_0/r_0$$

$$N_a = 5.2918 \times 10^{-11}/(6.60724 \times 10^{-1}\ ) \approx 80{,}000$$

$$a_0 = 80{,}000(r_0)$$

This places the electron at its lowest energy level in a hydrogen atom at a distance of 80,000 x r_0. We can also derive the Bohr radius using our relationship based on the Coulomb Barrier $E_R = E_C/N$

$$N = \frac{E_C}{E_R} \approx \frac{1.0897 \times 10^6 \text{ eV}}{13.5984 \text{ eV}} = 8.0134 \times 10^4 \approx 80{,}000$$

- $E_C = 1.0897 \text{ MeV} \approx 1.0897 \times 10^6 \text{ eV}$
- $E_R \approx 13.5984 \text{ eV} \approx 13.6 \text{ eV}$

This demonstrates that our equation based on Coulombs barrier also calculates the Bohr radius accurately.

$$R = N\,r_0 = 8.0134 \times 10^4 \times 6.60724 \times 10^{-16} \text{ m} \approx 5.29 \times 10^{-11} \text{ m}$$

Interestingly we can relate the two equations to derive the following relationship:

$$N_I = \frac{(6\, g_\Sigma)^2}{\pi\, \alpha^{-1}}$$

This is an alternate relationship based on g_Σ.

We can also relate the reaction shell number scales (n) to QWST wave geometry:

$$N(n) = \frac{E_C}{E_n} = \frac{E_C\, n^2}{13.6\ \text{eV}}, \quad \text{with} \quad N(1) = \frac{E_C}{13.6\ \text{eV}}.$$

Using $EC \approx 1.0897 \times 10^6\ eV$ we have:

$$N(1) \approx \frac{1.0897 \times 10^6\ \text{eV}}{13.6\ \text{eV}} \approx 8.0 \times 10^4,$$

Which provides us with a general relationship for n:

$$N(n) \approx 8.0 \times 10^4\, n^2$$

In summary, we are able to derive the Bohr radius from first principles within the QWST framework, demonstrating that the electron's ground state emerges naturally from the interplay of wave geometry, pressure, and energy.

VIII.3 – Ionization Energy (E_I)

Selected Investigation - Ionization Energy

We investigated whether we could calculate the Ionization Energy using QWST constants. We were able to derive a formula for E_I and verify its result. The new formula provides a compelling relationship to the energy density P_0 and constants related to wave geometry, r_0^3 , g_Σ^2 and A:

$$E_I = \frac{E_N}{108\, g_\Sigma^2} = \frac{A P_0 r_0^3}{72\, g_\Sigma^2}$$

We also derived an equation based on E_C

$$E_C = \frac{A\, P_0\, r_0^4}{\pi\,(2r_0)\,\alpha^{-1}} = \frac{A\, P_0\, r_0^3}{2\pi\, \alpha^{-1}}$$

As well as the relationship:

$$E_I = E_C / N_I \approx 13.6\, eV$$

This also yields the relationship for N_I in terms of g_Σ

$$N_I = \frac{(6\, g_\Sigma)^2}{\pi\, \alpha^{-1}}$$

Constants:

- $g_\Sigma = 980.665$ (Nucleon Stabilization Ratio)
- $r_0 = 6.60724 \times 10^{-16}$ m (nucleon radius, 1/4 wavelength of the basic frequency)
- $E_N = 2.25488815780(10^{-10})\, J$ (energy within the nucleon C-sphere)
- $P_0 = 5.15851475432(10^{35})\, Pa$ (the maximum stable energy density of quantum wavespace as well as within the nucleon C-sphere)
- $A = 1.010296164593 = \frac{16(\pi^2-8)}{3\pi^2}$ (wave geometry constant, simplifying placeholder)

Summary of Findings:

- We demonstrated that by starting from fundamental QWST constants—specifically, the maximum energy density P_0 and the nucleon's characteristic radius r_0—and employing the geometry of standing waves, we can derive the Coulomb barrier energy E_C (~ 1.1 MeV).

- We then derived an equation to determine the pressure within *even numbered* reaction shells at a distance defined by $R = N2r_0$, and further established that the energy at any reaction shell distance $R = N\,r_0$ scales according to $E_R = E_C/N$.

- By equating this derived reaction shell energy to the known electron ground-state energy (13.6 eV), we solved explicitly for N and successfully recovered the Bohr radius. This approach similarly allows for a direct calculation of ionization energy.

These equations show significant relationships between the Coulomb barrier, the Bohr radius, the number of wave shells N, and the nucleon stabilization ratio g_Σ. Additionally, with a method for calculating the pressure at specific reaction shells provides deeper insight into the nuclear-scale conditions that exist as nucleons approach fusion distances. These equations and derived relationships offer a valuable foundation for understanding fusion reactions through the wave-based geometric framework of QWST.

Grounded firmly in QWST's wave-based geometric framework, these equations open additional avenues toward fully understanding—and potentially overcoming—the challenges of achieving stable, controlled nuclear fusion. Further investigation, particularly through advanced computational modeling and systematic comparisons with empirical fusion data, may directly inform practical strategies and help accelerate fusion research.